AG codes over abelian surfaces containing no absolutely irreducible curves of low genus

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joint work with Yves Aubry, Fabien Herbaut, Marc Perret

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AGC^{2}T
(Gilles Lachaud Conference)
14/06/19

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Goppa codes

Let C be a *curve* of genus g and D a divisor on C.
Consider the Riemann-Roch space

$$L(D) = \{f \in \mathbb{F}_q(C) \setminus \{0\} \mid (f) + D \geq 0\} \cup \{0\}.$$

Definition:

Set $C(\mathbb{F}_q) = \{P_1, \ldots, P_n\}$. The code $\mathcal{C}(C, D)$ is defined to be the image of the evaluation map

$$\mathrm{ev} : L(D) \longrightarrow \mathbb{F}_q^n, \quad f \longmapsto (f(P_1), \ldots, f(P_n)).$$

$$\boxed{\begin{gathered} n = \#C(\mathbb{F}_q) \\ \dim(\mathcal{C}(C, D)) = \dim_{\mathbb{F}_q} L(D) \geq \deg D + 1 - g \\ d \geq n - \deg D \end{gathered}}$$

Basic notions and notations

$X/\mathbb{F}_q$ smooth, projective, absolutely irreducible algebraic surface

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- if C and D meet transversally then $C.D = \#(C \cap D)$
- symmetric
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Ample divisor: a divisor such that some multiple is a very ample divisor

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Ample divisor, the Nakai-Moishezon criterion:
H on X is ample $\iff H^2 > 0$ and $H.D > 0 \ \forall$ irreducible curve D on X

Evaluation codes

Let X be a surface and rH a very ample divisor on X.
Consider the Riemann-Roch space

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We consider $X = A$ to be an abelian surface and study $\mathcal{C}(A, rH)$.

Length, Dimension, Minimum Distance

$$n = ?$$

$$\dim(\mathcal{C}(A, rH)) = ?$$

$$d \geq ?$$

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dimension of the <u>Riemann-Roch space</u> $\Rightarrow$ <u>$\dim(\mathcal{C}(A, rH))$</u>

upper bound for <u>$N(f)$</u> $\Rightarrow$ lower bound for <u>the minimum distance</u>

A lower bound for the price of two upper bounds

Let $f \in L(rH) \setminus \{0\} = \{f \in \mathbb{F}_q(A) \setminus \{0\} \mid Z(f) - P(f) + rH \geq 0\}$.
We consider the effective divisor

$$D_f = rH + Z(f) - P(f) = \sum_{i=1}^{k} n_i D_i$$

where every D_i is an irreducible curve of arithmetic genus π_i and $n_i > 0$.
For $f \in L(rH) \setminus \{0\}$ we have

$$\boxed{\#Z(f) = N(f) \leq \sum_{i=1}^{k} \#D_i(\mathbb{F}_q)}$$

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$$\left.\begin{array}{c} \text{upper bound for } k \\ + \\ \text{upper bound for } \#D_i(\mathbb{F}_q) \end{array}\right\} \Rightarrow \text{lower bound for } d$$

Some tools on abelian surfaces

- Riemann-Roch Theorem:
  $\dim_{\mathbb{F}_q} L(rH) + \dim_{\mathbb{F}_q} L(K - rH) = \frac{1}{2} rH.(rH - K) + 1 + p_a + s(rH)$

Some tools on abelian surfaces

- Riemann-Roch Theorem:
  $\dim_{\mathbb{F}_q} L(rH) + \dim_{\mathbb{F}_q} L(-rH) = \frac{1}{2}r^2H^2 + s(rH)$

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- Rational Points: set $m := \lfloor 2\sqrt{q} \rfloor$
  - for D an irreducible curve on A of arithmetic genus π we have

$$\#D(\mathbb{F}_q) \leq q + 1 - \mathrm{Tr}(A) + |\pi - 2| m$$

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upper bound for $\underline{N(f)}$ $\Rightarrow$ lower bound for <u>the minimum distance</u>

Bound for $N(f)$

Let A be a simple abelian surface such that every absolutely irreducible curve on it has arithmetic genus $\pi > \ell$, for a positive integer ℓ.

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$$N(f) \leq \sum_{i=1}^{k} \# D_i(\mathbb{F}_q)$$

Write $k = k_1 + k_2$ where

$$k_1 = \#\{D_i \mid \pi_i > \ell\}$$

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$$N(f) \leq k_1(q + 1 - \mathrm{Tr}(A) - 2m) + m \sum_{i=1}^{k_1} \pi_i + k_2(\ell - 1)$$

Bound for $N(f)$

Lemma:

1. $k_2 \le r\sqrt{\frac{H^2}{2}} - k_1\sqrt{\ell}$,
2. $k_1\sqrt{\ell} \le r\sqrt{\frac{H^2}{2}}$,
3. $\sum_{i=1}^{k_1} \pi_i \le \left(r\sqrt{H^2/2} - k_1\sqrt{\ell}\right)^2 + r\sqrt{2H^2\ell} + (1-\ell)k_1$.

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$$N(f) \le \phi(k_1),$$

$$\phi(k_1) := m\ell k_1^2 + k_1\left(q + 1 - \mathrm{Tr}(A) - m(\ell+1) - mr\sqrt{2H^2\ell} - \sqrt{\ell}(\ell-1)\right)$$

$$+mH^2r^2/2 + mr\sqrt{2H^2\ell} + r\sqrt{H^2/2}(\ell-1)$$

and $k_1 \in \left[1, \sqrt{\frac{H^2}{2\ell}}r\right]$.

Bound for the minimum distance

We have:

$$N(f) \leq \begin{cases} \phi\left(\sqrt{\frac{H^2}{2\ell}}r\right) \text{ if } \sqrt{\frac{2\ell}{H^2}} \leq r \leq \frac{\sqrt{2}(q+1-\mathrm{Tr}(A)-m-\sqrt{\ell}(\ell-1))}{m\sqrt{H^2\ell}}, \\ \phi(1) \text{ otherwise.} \end{cases}$$

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Recall that

$$d = \#A(\mathbb{F}_q) - \max_{f\in L(rH)\setminus\{0\}} N(f)$$

Bound for the minimum distance

Theorem: (Aubry, B., Herbaut, Perret)
Let A be a simple abelian surface of trace $\mathrm{Tr}(A)$ such that every irreducible curve on it has arithmetic genus $\pi > \ell$, for a positive integer ℓ. Then the minimum distance d of the code $\mathcal{C}(A, rH)$ satisfies:

$$\boxed{d \geq \#A(\mathbb{F}_q) - r\sqrt{\frac{H^2}{2\ell}}\,(q + 1 - \mathrm{Tr}(A) + (\ell - 1)m)}$$

if $\sqrt{\frac{2\ell}{H^2}} \leq r \leq \frac{\sqrt{2}(q+1-\mathrm{Tr}(A)-m-\sqrt{\ell}(\ell-1))}{m\sqrt{H^2\ell}}$, *otherwise*

$$\boxed{d \geq \#A(\mathbb{F}_q) - (q + 1 - \mathrm{Tr}(A)) - m(r^2H^2/2 - 1) - r\sqrt{\frac{H^2}{2}}(\ell - 1).}$$

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$$d_{min} - \#A(\mathbb{F}_q) \underset{q\to\infty}{\sim} -r\sqrt{\frac{H^2}{2\ell}}q.$$

Remark: the bound for $\ell = 2$ is better than the one for $\ell = 1$!

Question: There exist abelian surfaces which do not contain absolutely irreducible curves of arithmetic genus 0, 1 nor 2?

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YES!

Abelian surfaces without curves of low genus: starting point

Lemma:

An abelian surface A contains no absolutely irreducible curves of arithmetic genus 0, 1 *nor* 2 $\iff$ *A is simple and not isogenous to a Jacobian surface.*

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Theorem: (Weil)

Let (A, λ) be a principally polarized abelian surface defined over the finite field k. Then (A, λ) is either

1. *the polarized Jacobian of a genus 2 curve over k,*
2. *the product of two polarized elliptic curves over k,*
3. *the Weil restriction of a polarized elliptic curves over a quadratic extension of k.*

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Abelian surfaces that might have the property we are searching for:

- Weil restrictions of polarized elliptic curves over a quadratic extension of k,
- abelian surfaces defined over k that do not admit a principal polarization.

Abelian surfaces containing no curves of genus 0, 1 nor 2

Proposition: (Aubry, B., Herbaut, Perret)

(i) *Let A be an abelian surface defined over $\mathbb{F}_q$ which does not admit a principal polarization. Then A does not contain absolutely irreducible curves of arithmetic genus* 0, 1 *nor* 2.

(ii) *Let $q = p^e$. Let E be and elliptic curve defined over $\mathbb{F}_{q^2}$ of trace $\mathrm{Tr}(E/\mathbb{F}_{q^2})$. Let A be the $\mathbb{F}_{q^2}/\mathbb{F}_q$-Weil restriction of the elliptic curve E. Then A does not contain absolutely irreducible curves defined over $\mathbb{F}_q$ of arithmetic genus* 0, 1 *nor* 2 *if and only if one of the following cases holds:*

(1) $\mathrm{Tr}(E/\mathbb{F}_{q^2}) = 2q - 1$;
(2) $p > 2$ *and* $\mathrm{Tr}(E/\mathbb{F}_{q^2}) = 2q - 2$;
(3) $p \equiv 11 \mod 12$ *or* $p = 3$, $q = \square$ *and* $\mathrm{Tr}(E/\mathbb{F}_{q^2}) = q$;
(4) $p = 2$, $q \neq \square$ *and* $\mathrm{Tr}(E/\mathbb{F}_{q^2}) = q$;
(5) $q = 2$ *or* $q = 3$, *and* $\mathrm{Tr}(E/\mathbb{F}_{q^2}) = 2q$.

What's next? Some ideas...

I) Genus 3 curves. There exist surfaces which do not contain absolutely irreducible genus 3 curves as well? If so, under which condition(s)?

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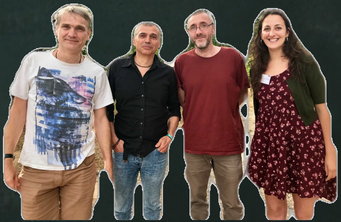
Work in progress...with the same old team (MYFE)



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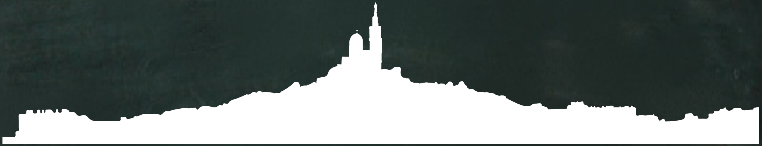
Thank you for your attention!

(Questions?)



He who asks a question is a fool for five minutes;
he who does not ask a question remains a fool forever.

Confucius