On the number of rational points of curves over a surface in $\mathbb{P}^3$

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(Joint work with Jade Nardi)

The number of rational points on a smooth projective absolutely irreducible curve C of genus g defined over the finite field $\mathbb{F}_q$ is bounded by the famous Serre–Weil bound, namely $\#C(\mathbb{F}_q) \leq q+1+g\lfloor 2\sqrt{q}\rfloor$. Several works have been devoted to improve this bound for a range of parameters, and to extend it to more general curves, possibly reducible or singular [4, 1, 3].

In this talk, we will show that the number of rational points on an irreducible curve of degree δ defined over a finite field $\mathbb{F}_q$ lying on a surface S in $\mathbb{P}^3$ of degree d is, under certain conditions, bounded by $\delta(d+q-1)/2$. Within a certain range of δ and q, this result improves all other known bounds in the context of space curves. The method we used is inspired by techniques developed by Stöhr and Voloch [4]. In their seminal work of 1986, they introduced the Frobenius orders of a projective curve and used them to give an upper bound on the number of rational points of the curve. After recalling some general results on the theory of orders of a space curve, we will study the arithmetic properties of curves lying on a surface in $\mathbb{P}^3$, to finally prove the bound.

The talk is based on the preprint [2].

References

[1] Yves Aubry and Marc Perret. *A Weil theorem for singular curves.* pages 1–8. De Gruyter, 1993.

[2] Elena Berardini and Jade Nardi. *Curves on Frobenius classical surfaces in $\mathbb{P}^3$ over finite fields.* arXiv preprint arXiv:2111.09578, 2021.

[3] Masaaki Homma. *A bound on the number of points of a curve in a projective space over a finite field.* Theory and applications of finite fields, volume 579 of Contemp. Math., pages 103 – 110. Amer. Math. Soc., Providence, RI, 2012.

[4] Karl-Otto Stöhr and José Felipe Voloch. *Weierstrass Points and Curves Over Finite Fields.* Proceedings of The London Mathematical Society, 1, 1–19, 1986.